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THE EQUIVALENCE PRINCIPLE ON THE BASIS OF FIELD DEPENDENT LORENTZ TRANSFORMATION

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ABSTRACT

The equality of gravitational and inertial mass within the framework of equivalence principle was investigated. The gravitational mass is expressed in terms of speed and potential by using velocity invariance and curved space Lorentz transformation. The same transformations were used to express inertial mass in terms of acceleration and potential. The gravitational mass and the inertial one are shown to be equal.

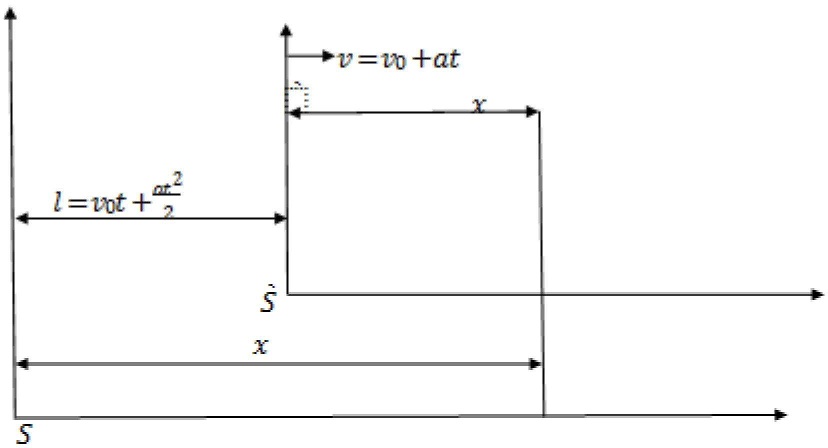
Keywords: *Equivalence Principle; velocity Invariance; Lorentz Transformation; Curved Space.*

I. INTRODUCTION

Equivalence principle is related to inertia. Equivalence principle emerged when Galileo found that the acceleration of a mass due to gravitation is independent of the amount of mass being accelerated. Galileo obtained his results with balls rolling down nearly frictionless inclined planes to slow the motion and increase the timing accuracy. Increasingly precise experiments have been performed by Loránd Eötvös using the torsion balance pendulum [1]. From Newton second law one can estimate the inertial mass due to force which accelerate the body, and the gravitation mass can be estimated from Newton law of gravity. Albert Einstein developed his general theory of relativity starting from the assumption that this correspondence between inertial and gravitational mass is not accidental. However, in his theory, gravitation is not a force and thus not subjects to Newton's third law, so the equality of inertial and gravitational mass remains as puzzling [2]. Einstein also referred to two reference frames, S and $\grave{S}$. S is a uniform gravitational field, whereas $\grave{S}$ is uniformly accelerated such that objects in the two frames experience identical forces. Einstein combined the equivalence principle with special relativity to predict that time is affected by a gravitational potential, and light rays bend in a gravitational field, before he developed the concept of curved space-time. So the original equivalence principle, as described by Einstein, predicted that freefall and inertial motion were physically equivalent. Inertial mass is the mass of an object measured by its resistance to acceleration. This definition has been championed by Ernst Mach and has been developed by Percy W. Bridgman [3,6]. In special relativity, there are two kinds of mass: rest mass and relativistic mass, which increases with velocity. Rest mass is the Newtonian mass as measured by an observer at rest. Relativistic mass is proportional to the total quantity of energy in a body or system. But there is no term refers to the effect of gravity field or any fields on mass. This work is concerned with the effect of gravity field and acceleration on the mass. Then one can verify equivalence principle.

II. THE EQUIVALENCE PRINCIPLE ON THE BASIS OF AVERAGE VELOCITY LORENTZ TRANSFORMATION

The equivalence principle states the laws of nature takes the same form in accelerated frame and the frame permeated by gravitational field. This can be studied here with in frame work of lorentz transformation based on average velocity assumption as proposed by M.Dirar and other[7]. In this version the relativistic mass is given by



$$m = \frac{m_0}{\sqrt{1 - \frac{v_m^2}{c^2}}} \quad (1)$$

Where the average velocity which was assumed to be invariant

$$v_m = \frac{v_0 + v}{2} \, (2)$$

For particle moving with initial velocity v_0 against gravity, the final velocity is given according to the ordinary rectilinear motion with constant acceleration to be

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\varphi \quad (3)$$

Where φ stands for the potential per unit mass. Thus according to equation (3), (2) and (1)

$$v_m^2 = \frac{v_0^2 + 2vv_0 + v^2}{4} = \frac{2v^2 + 2\varphi + 2v\sqrt{v^2 + 2\varphi}}{4}$$
$$= \frac{v^2 + \varphi + v\sqrt{v^2 + 2\varphi}}{2} \quad (4)$$

Thus equation (1) become

$$m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - \left(\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2}\right)}} \quad (5)$$

Consider now a particle falling in a gravitational field such that its speed at a certain point x is v and its potintial energy is φ. According to equation (5) its mass is given by

$$m_g = m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - \left(\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2}\right)}} \quad (6)$$

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Which stands for gravitational mass. Now consider a particle moving upward with speed v_0. For an elevator moving with acceleration a up ward its velocity after time t when its displacement is x becomes

$$v^2 = v_0^2 - 2ax \quad (7)$$

Thus according to equations (1), (2) and (7) one gets

$$v_m^2 = \frac{v_0^2 + 2vv_0 + v^2}{4}$$

$$v_m^2 = \frac{v^2 + ax + v\sqrt{v^2 + 2ax}}{2}$$

Thus equation (1) gives the inertial mass $m_i$is gives

$$m_i = m = \frac{m_0}{\sqrt{1 - \frac{ax}{2c^2} - \left(\frac{v^2+v\sqrt{v^2+2ax}}{2c^2}\right)}} \quad (8)$$

But numerically

$$ax = \varphi \quad (9)$$

Thus

$$m_i = m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - \left(\frac{v^2+v\sqrt{v^2+2\varphi}}{2c^2}\right)}} \quad (10)$$

A direct comparison of equations (6) and (10) gives

$$m_g = m_i \quad (11)$$

This means that the inertial and gravitational masses are equivalent. Another version of equivalence principle states that the law of nature for elevator in free fall in gravitational fields takes the same form as that in free space. Now consider an elevator which is in free fall with a particle of rest mass m_0 inside it. According to equation (2)

$$v_m = \frac{v_0 + v}{2}$$

Since the particle is at rest all times, it follows that

$$v_0 = 0 \ \& \ v = 0 \quad (12)$$

Thus

$$v_m = 0 \quad (13)$$

Thus in view of equation (1) the gravitational mass is

$$m_g = \frac{m_0}{\sqrt{1-0}} = m_0 \quad (14)$$

For a particle of rest mass inside an elevator in free space that have rest mass m_0:

$$v_0 = 0 \ \& \ v = 0 \ \& v_m = 0 \quad (15)$$

Thus according to equation (1) the inertial mass is given by

$$m_i = \frac{m_0}{\sqrt{1-0}} = m_0 \quad (16)$$

III. INERTIAL AND GRAVITATIONAL MASS IN A CURVED SPACE LORENTZ TRANSFORMATION

The equation of motion of an accelerated particle due the action of gravity field or due to the elevator acceleration a with respect to a particle in free space is given by

$$\frac{d^2x^\lambda}{dt^2}+\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}=0 \quad (17)$$

For slow moving particle in x-direction in a weak gravitational field, equation (17) is reduced to

$$\frac{d^2x}{dt^2}+c^2\Gamma^1_{00}=0 \quad (18)$$

The connection in a weak field limit is given by

$$\Gamma^1_{00}=-\frac{1}{2}\nabla h_{00} \quad (19)$$

Thus the acceleration a can curve the space according to equation (18) and (19) to get

$$a=\frac{d^2x}{dt^2}=-c^2\Gamma^1_{00}=\frac{c^2}{2}\nabla h_{00}=\frac{c^2}{2}\frac{\partial h_{00}}{\partial x} \quad (20)$$

For constant acceleration

$$\int adx=\frac{c^2}{2}\int dh_{00}$$
$$2ax=c^2h_{00} \quad (21)$$

By using the definition of force therefore,

$$F=ma=-\nabla V=-m\nabla\varphi=-\frac{mc^2}{2}\nabla h_{00} \quad (22)$$

Thus

$$\varphi=-\frac{c^2}{2}h_{00}\ \&\ h_{00}=-\frac{2\varphi}{c^2} \quad (23)$$

Therefore, the time metric takes the form

$$g_{00}=-1+h_{00}=-\left(1+\frac{2\varphi}{c^2}\right) \quad (24)$$

In view of equation (21) and (24) it also takes the form

$$g_{00}=-1+h_{00}=-1+\frac{2ax}{c^2}=-\left(1-\frac{2ax}{c^2}\right) \quad (25)$$

Taking into account equation (21) and (23) one gets

$$\frac{2\varphi}{c^2}=-\frac{2ax}{c^2} \quad (26)$$

The mass in a curved space takes the form [8]

$$m=\frac{m_0}{\sqrt{-g_{00}-\frac{v^2}{c^2}}} \quad (27)$$

Using equations (27) and (24) the gravitational mass is given by

$$m_g = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{\left(1+\frac{2\varphi}{c^2}\right) - \frac{v^2}{c^2}}} \quad (28)$$

With the aid of equation (27) and (25) the inertial mass is given by

$$m_i = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{2ax}{c^2} - \frac{v^2}{c^2}}} \quad (29)$$

Taking into account equations (25), (24); it follows that

$$g_{00} = -\left(1+\frac{2\varphi}{c^2}\right) = -\left(1-\frac{2ax}{c^2}\right) \quad (30)$$

Therefore

$$m_g = m_i \quad (31)$$

i.e. the inertial and gravitational mass are equal. For an elevator freely falling with a particle in a gravitational field, the particle is at rest with respect to him. Thus no acceleration is observed, i.e.

$$v = 0 \quad \& \quad a = 0 \quad (32)$$

According to equation (30) and (28)

$$m_g = m_0 \quad (33)$$

For the particle in free space, which is at rest with respect to an elevator

$$v = 0 \quad \& \quad a = 0 \quad (34)$$

Again equation (30) and (29) gives

$$m_i = m_0 \quad (35)$$

Thus

$$m_g = m_i \quad (36)$$

IV. RESULTS AND DISCUSSION

The equality of gravitational and inertial mass is studied with in the frame work of velocity invariance and curved space Lorentz transformation. Two sinarios are proposed. In the first approach the particle mass falling freely in a gravitational field is compared with the one in free space observed by an observer in an elevator moving with respect to him with acceleration equal to the gravity acceleration. According to this version the velocity invariant Lorentz transformation shows the equality of gravitational mass and inertial mass as shown by equation (11). This equality is related to the equality of potential per unit mass and work done due to acceleration [see equation (9)]. The curved space Lorentz transformation shows also equality of gravity and inertial mass [see equation (31)]. This is due to the fact that both field and acceleration deform the space as shown by equation (30). In the second version the elevator falling with a particle in a gravitational field is compared with that in free space at rest with respect to the particle. In velocity invariant Lorentz transformation gravitational mass is equal to the rest mass [see equation (14) and (16)]. In the curved space Lorentz transformation, the fact that the particle is at rest in both make acceleration vanishes. Thus the space is Minkowskian and the inertial and gravitational mass are equal to rest mass in both frames [see equations (32) -(36)]

V. CONCLUSION

The velocity invariant model and curved space Lorentz transformation shows the equality of gravitational and inertial mass. This equality is related to the equality of potential and acceleration work done per unit mass

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